NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1241

EXPERIMENTAL INVESTIGATION OF THE STRESS DISTRIBUTION AROUND REINFORCED CIRCULAR CUT-OUTS IN SKIN-STRINGER PANELS UNDER AXIAL LOADS

By Daniel Farb

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Washington

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SUMMARY

Results are presented of strain surveys around four reinforced circular cut-outs successively made in an axially loaded skin-stringer tension panel. These surveys were made in the elastic range. In each case the reinforcing rings were of rectangular cross section, and the cross-sectional area of the reinforcement approximately equaled the cross-sectional area of the skin and stringers removed at the transverse center line of the cut-out.

Curves are presented which show the distribution of stringer and shear stresses in the panel and bending stresses in the reinforcing rings. Empirical methods are given for estimating the maximum stringer stress in the panel and for approximating the longitudinal stresses in the stringers and rings at the transverse center line of the cut-out.

INTRODUCTION

As a result of a lack of analytical and experimental information, the design of reinforced circular cut-outs in skin-stringer combinations at present depends largely upon judgment and experience. This paper gives experimental information which may be used to determine the suitability of present design practices and should be useful for comparison with the results of future analyses.

Strain surveys in the elastic range were made around reinforced circular cut-outs in a flat skin-stringer panel. The rings reinforcing the cut-outs were of rectangular cross section, and the cross-sectional area of reinforcement approximately equaled the cross-sectional area of skin and stringers removed at the transverse center line of the cut-out. Curves are presented to show the

experimental distributions of stringer and shear stresses in the panel and the bending stresses in the rings. Empirical relationships are presented to enable the designer to approximate quickly the maximum stringer stress in the panel and the stringer stresses and longitudinal ring stresses at the transverse center line of the cut-out.

TEST SPECIMENS AND PROCEDURE

The test specimen was a 16-stringer 24S-T aluminum alloy panel 144 inches long and approximately 48 inches wide, with a cross section as shown in figure 1. A schematic diagram of the panel is shown in figure 2.

Four circular cut-outs of progressively increasing diameter were made at the center of the panel. Each was tangent to continuous stringers at the transverse center line. The cut-outs had diameters of approximately 10, 22, 34, and 46 inches; for the smallest diameter, only the two center stringers of the panel were cut, and for the largest diameter only the two outside stringers remained uncut. Each cut-out was reinforced by a pair of circular rings of rectangular cross section, one on each side of the panel. Ring dimensions are given in the following table:

Stringers cut	Inside radius (in.)	Outside radius (in.)	Median (in.)	Thickness of ring (in.)
2	4.31	5.07	4.69	0.25
6	10.06	11.82	10.94	.25
10	15.81	18.57	17.19	.25
14	21.55	25.31	23.43	.25

The rings were riveted to the sheet as well as to the stringers. Filler plates were placed on the sheet under the rings and were fastened in such a manner as to be as ineffective as possible in carrying load. In each case at the transverse center line of the cut-out the cross-sectional area of the rings approximately equaled the cross-sectional area of the skin and stringers removed.

Shear and stringer stresses in the panel as well as bending
stresses in the reinforcing rings were experimentally determined
for each cut-out condition. Before the first cut-out was made,
tests were conducted to determine the elastic properties of the
panel. An effective value of Young's modulus of 10.43×10^3 ksi
was derived from measurements of the strains in all stringers at
three stations along the span. The effective modulus may be
considered as including corrections for the effect of rivet holes,
average gage calibration factor, and dynamometer calibration factor.
The average transverse strain indicated an effective Poisson's
ratio of 0.302. The effective shear modulus was found to be
4.01×10^3 ksi. The preceding effective moduli were used to convert
measured strain into stress.

The test setup is shown in figure 3. The total external load
was uniformly distributed to the ends of the stringers by means of
whippletrees. Stringer and shear strains were measured by Tuckerman
optical strain gages with gage lengths of 2 inches. The shear
strains in the sheet were obtained from strains measured at angles
of 45° and 135° to the direction of the stringers. The gages were
used in pairs, one gage on each side of the test panel. Strains
were measured at corresponding points in all four quadrants of the
doubly symmetrical panel, and the final figures are drawn as for one
quadrant. Each plotted point represents, therefore, the average of
four stations. Bending stresses in the rings were determined from
resistance-type wire-strain-gage measurements. A typical arrangement
of wire strain gages on the rings is shown in figure 4. The gages
were placed on the inner and outer edges of each ring at regular
intervals along the perimeters. The plotted experimental values
represent the average stresses in each pair of rings reinforcing a
cut-out.

The load was applied in three equal increments. A load-stress
plot was made for each gage, and a straight line was drawn through
the plotted points. If the line through the plotted points did not
pass through the origin, a line parallel to it was drawn through the
origin. The line through the origin was used to determine the
measured strain at maximum load. If the necessary shift was more
than 0.2 ksi, however, a new set of readings was taken.

RESULTS AND DISCUSSION

Figures 5 to 8 show the spanwise distribution of experimental
stringer stresses. It may be noted from the curves that the maximum

stringer stress for each cut-out condition occurred in either the
continuous stringer bounding the cut-out or in the cut stringer
adjacent to this continuous stringer, and that, in general, the
maximum stresses in both stringers were approximately equal. The
maximum stress occurred at a transverse station which intersected
the median line of the reinforcing rings at approximately 45°
from the transverse center line.

Chordwise distributions of stringer stresses are shown in
figure 9. At a distance of one radius from the transverse center
line, the chordwise stress distribution was nearly uniform from
the outer edge of the panel to some point close to the first cut
stringer and then decreased approximately linearly to zero near
the longitudinal center line. At the transverse center line the
chordwise distribution was approximately uniform except in those
panels with few uncut stringers.

The spanwise-distribution curves for the experimental shear
stresses are shown in figures 10 to 13. Chordwise distributions
of shear stresses are shown in figure 14.

The distribution of bending stresses in the reinforcing rings
is plotted in figure 15 for each cut-out condition. It has been
suggested in estimating these bending stresses that, the stringer
loads which existed at a large distance from the cut-out should be
applied directly to the rings, without allowing for a redistribution
of the forces through the skin-stringer combination. The maximum
stresses resulting from this type of analysis were computed for
the four cut-out conditions tested and were compared with the maximum
bending stresses computed from the test data. In these calculations
only the actual area of the rings was used to determine the moment
of inertia. The calculations were made for a load of 15 kips. The
results of this comparison are given in the following table. It
may be observed that for the four cut-outs under consideration
this method is unduly conservative.

Stringers cut	Maximum bending stress (ksi)	
	Experimental	Calculated
2	2.8	29.6
6	3.0	34.6
10	4.7	41.2
14	12.0	32.5

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Strain surveys in the elastic range were made around reinforced circular cut-outs in a flat skin-stringer panel. The rings reinforcing the cut-outs were of rectangular cross section, and the cross-sectional area of reinforcement approximately equaled the cross-sectional area of skin and stringers removed at the transverse center line of the cut-out. Curves are presented to show the

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Shear and stringer stresses in the panel as well as bending stresses in the reinforcing rings were experimentally determined for each cut-out condition. Before the first cut-out was made, tests were conducted to determine the elastic properties of the panel. An effective value of Young's modulus of 10.43×10^3 ksi was derived from measurements of the strains in all stringers at three stations along the span. The effective modulus may be considered as including corrections for the effect of rivet holes, average gage calibration factor, and dynamometer calibration factor. The average transverse strain indicated an effective Poisson's ratio of 0.302. The effective shear modulus was found to be 4.01×10^3 ksi. The preceding effective moduli were used to convert measured strain into stress.

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stringer stress for each cut-out condition occurred in either the
continuous stringer bounding the cut-out or in the cut stringer
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figure 9. At a distance of one radius from the transverse center
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the four cut-out conditions tested and were compared with the maximum
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only the actual area of the rings was used to determine the moment
of inertia. The calculations were made for a load of 15 kips. The
results of this comparison are given in the following table. It
may be observed that for the four cut-outs under consideration
this method is unduly conservative.

Stringers cut	Maximum bending stress (ksi)	
	Experimental	Calculated
2	2.8	29.6
6	3.0	34.6
10	4.7	41.2
14	12.0	32.5

The strain measurements on the reinforcing rings indicated that at the transverse center line the average longitudinal stress in the rings was approximately 58 percent of the average stress in the skin and stringers. Since the chordwise distribution of experimental stringer stress was found to be approximately uniform, the stringer stresses and the longitudinal stresses in the rings at the transverse center line may be approximated in the following manner:

$$\sigma_s = \frac{P}{A_s + 0.58 \, A_r} \qquad (1)$$

$$\sigma_r = 0.58 \, \sigma_s \qquad (2)$$

where σ_s is the average stringer stress, P is the external load on the panel, A_s is the cross-sectional area of skin and stringers, A_r is the cross-sectional area of rings, and σ_r is the average longitudinal stress in the rings. The experimental average stringer stresses and the average stringer stresses calculated by the empirical relationship of equation (1) at the transverse center line for a load of 15 kips are given in the following table:

Stringers cut	Experimental σ_s (ksi)	Calculated σ_s (ksi)
2	3.9	3.9
6	4.5	4.4
10	4.6	5.1
14	6.1	5.9

The experimental longitudinal stresses and the longitudinal stresses calculated by the empirical relationship of equation (2) in the reinforcing rings at the transverse center line are given in the following table for a load of 15 kips:

Stringers cut	Experimental σ_r (ksi)	Calculated σ_r (ksi)
2	2.6	2.3
6	2.5	2.6
10	3.1	3.0
14	3.2	3.4

The maximum stringer stresses may be approximated by the empirical formula

$$\sigma_{max} = \left(1 + \frac{D}{b}\right)\sigma_s \qquad (3)$$

where σ_{max} is the maximum stringer stress in the panel, D is the median diameter of the reinforcing rings, b is the width of the panel, and σ_s is the average stringer stress from equation (1).
The experimental maximum stringer stresses and the maximum stringer stresses calculated by the empirical relationship of equation (3) for a load of 15 kips are given in the following table:

Stringers cut	Experimental σ_{max} (ksi)	Calculated σ_{max} (ksi)
2	5.1	4.7
6	6.2	6.4
10	8.4	8.8
14	13.0	11.7

The empirical relationships should be applied only to cut-outs with ring reinforcements similar to those of the test panel.

An analysis was made by means of the numerical procedure of reference 1, but it was discovered that because of the complex interaction of the reinforcing rings with the skin-stringer combination near the cut-out, an impracticable number of displacements would have to be calculated before satisfactory results could be obtained.

CONCLUSIONS

Results were presented of strain surveys around four reinforced circular cut-outs of varying diameter successively made in an axially loaded skin-stringer panel. These surveys were made in the elastic range and showed in the form of curves the distribution of stringer and shear stresses in the panel and bending stresses in the reinforcing rings. In each case the reinforcing rings were of rectangular cross section, and the cross-sectional area of the reinforcement approximately equaled that of the panel removed at the transverse center line of the cut-out. The following conclusions were indicated:

1. The maximum stringer stress occurred at a transverse station which intersected the median line of the reinforcing rings at approximately 45° from the transverse center line of the cut-out. The maximum stringer stress occurred in either the continuous stringer bounding the cut-out or in the cut stringer adjacent to this continuous stringer.

2. In estimating the bending stresses in the reinforcing rings, the practice of assuming the stringer loads at a large distance from the cut-out applied directly to the rings was found to be unduly conservative.

3. Empirical relationships were determined for evaluating the maximum stringer stress in the panel and for approximating the longitudinal stresses in the stringers and rings at the transverse center line of the cut-out.

Langley Memorial Aeronautical Laboratory
     National Advisory Committee for Aeronautics
          Langley Field, Va., Dec. 27, 1946

REFERENCE

1. Kempner, Joseph: Application of a Numerical Procedure to Stress Analysis of Stringer-Reinforced Panels. NACA ARR No. L5C09a, 1945.

.1002   .0322   ← .754

ℓ panel

7 @ 3.125 = 21.88   ←1.563
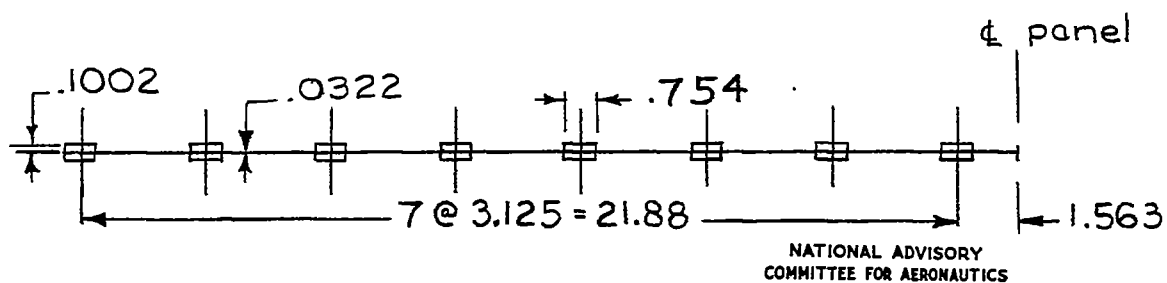
Figure 1.- Cross section of test panel.

72.0

Stringer 1

Bay 1

2   2

3   3

4   4

5   —5

6   6

7   7

8   8

4.69

10.94

17.19

23.43

—8
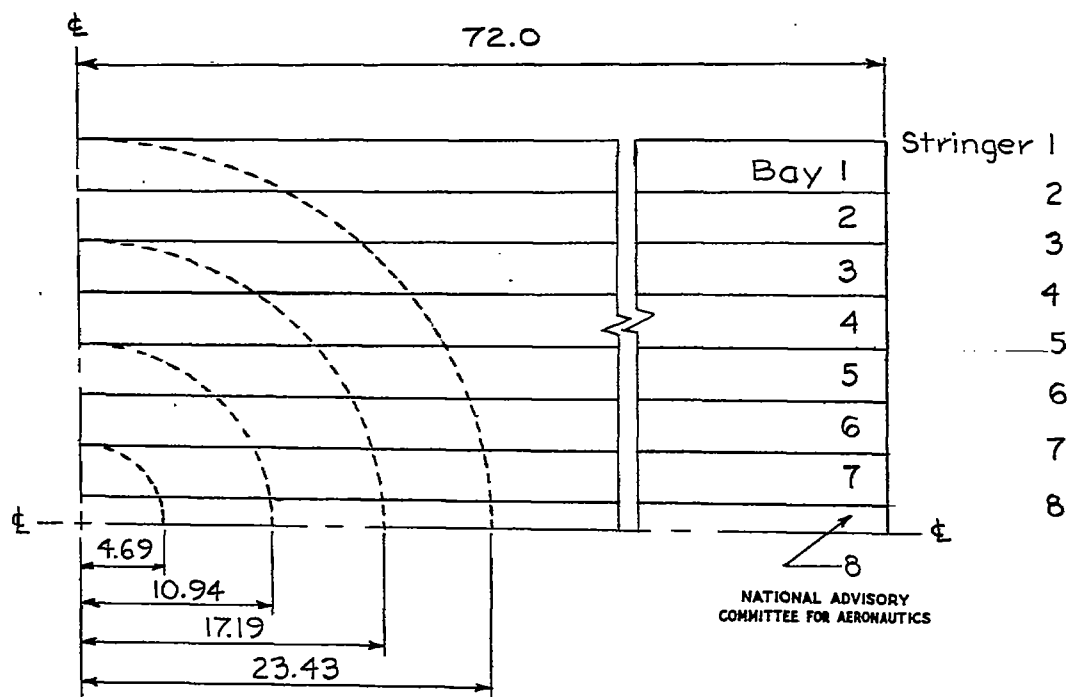
Figure 2.- Schematic diagram of one quadrant of test panel. Dashed
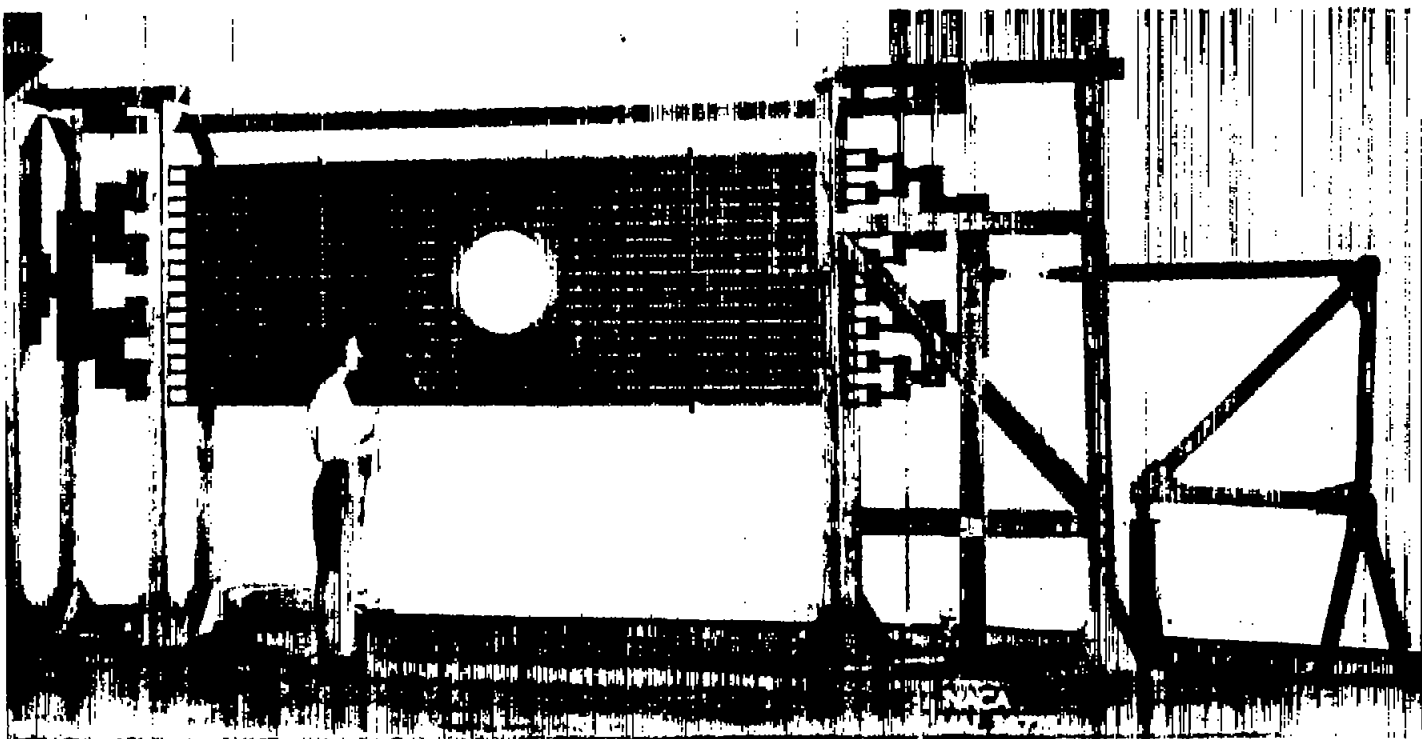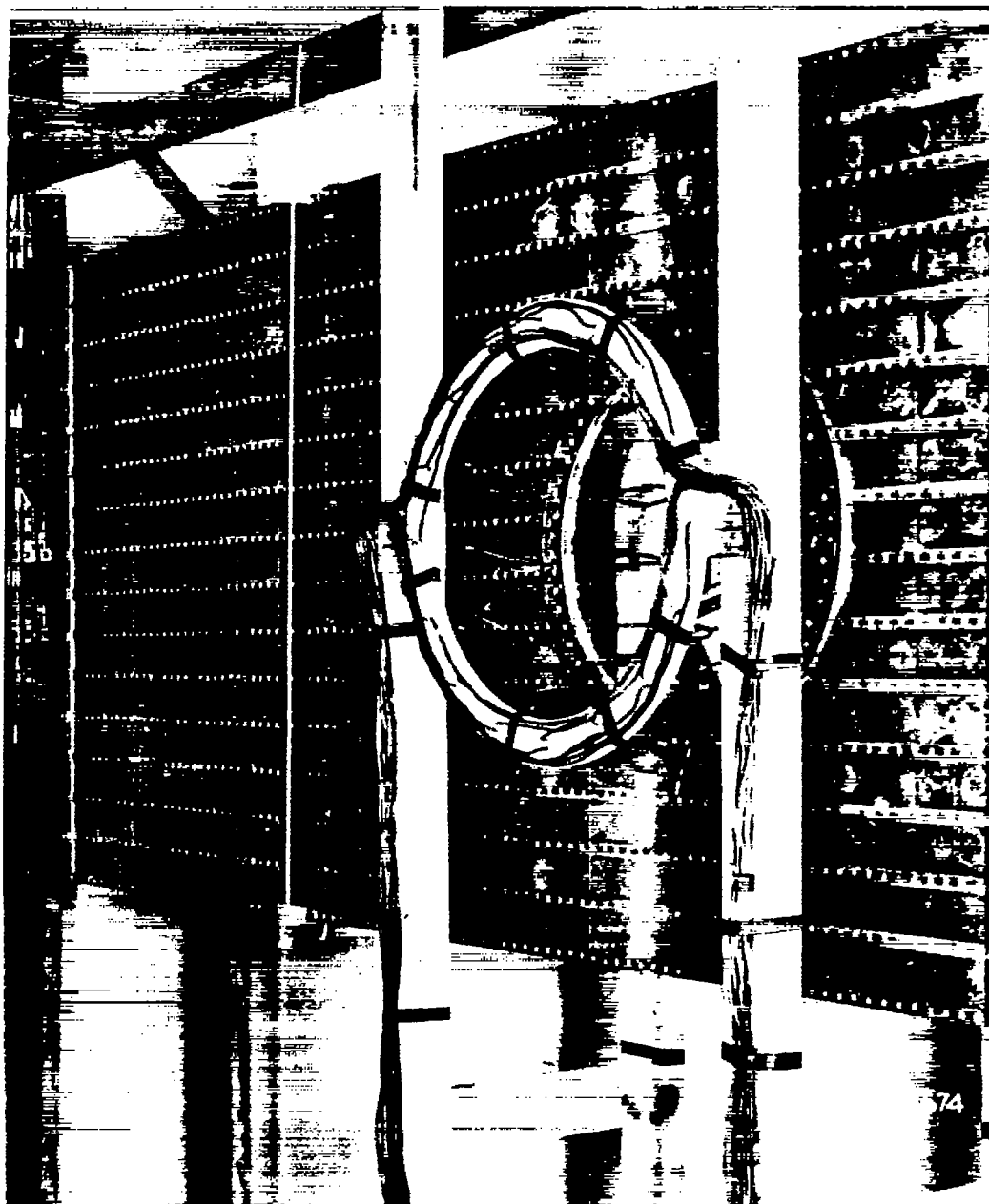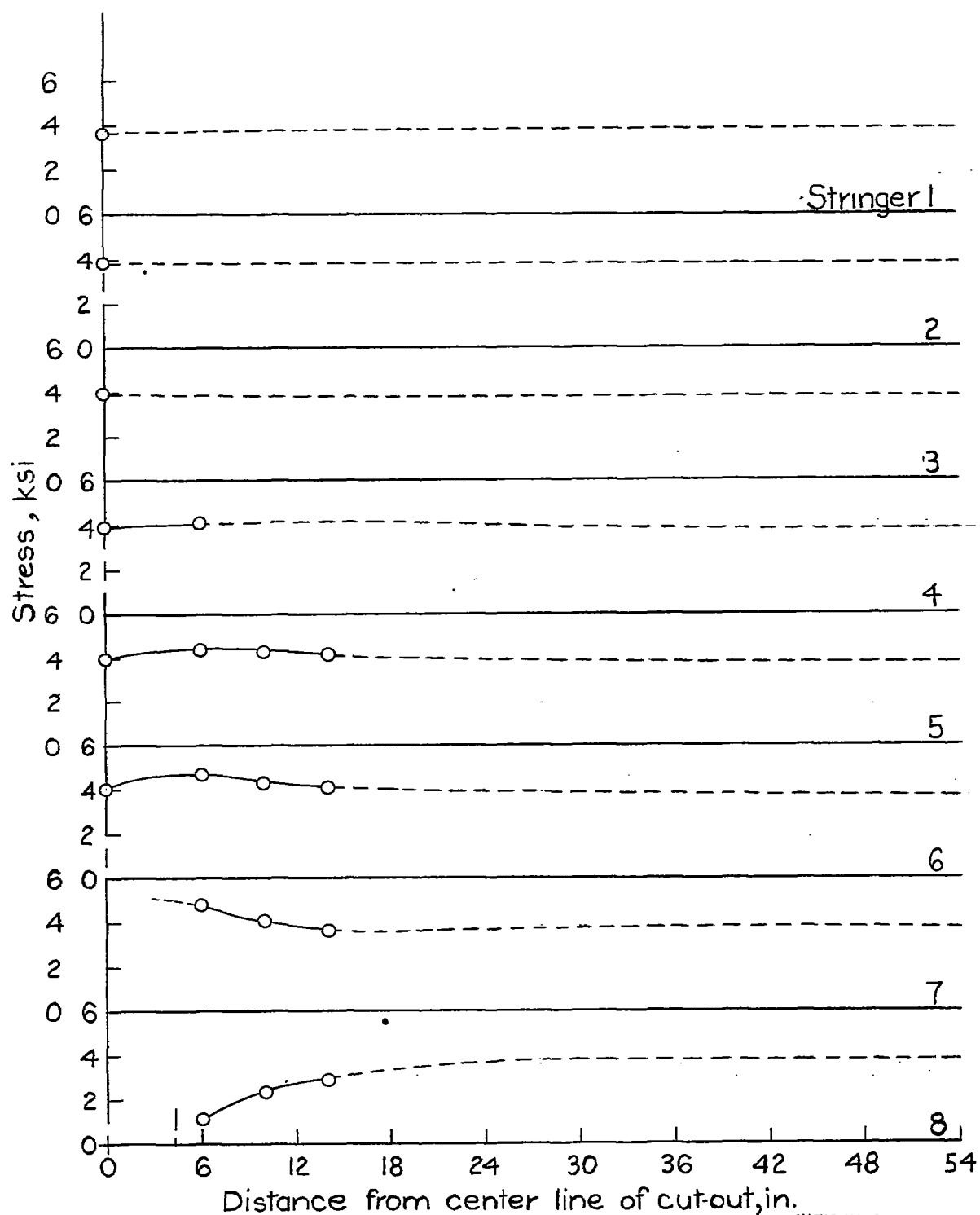lines indicate median lines of reinforcing rings.

Figure 3.- Test set-up.

Fig. 3

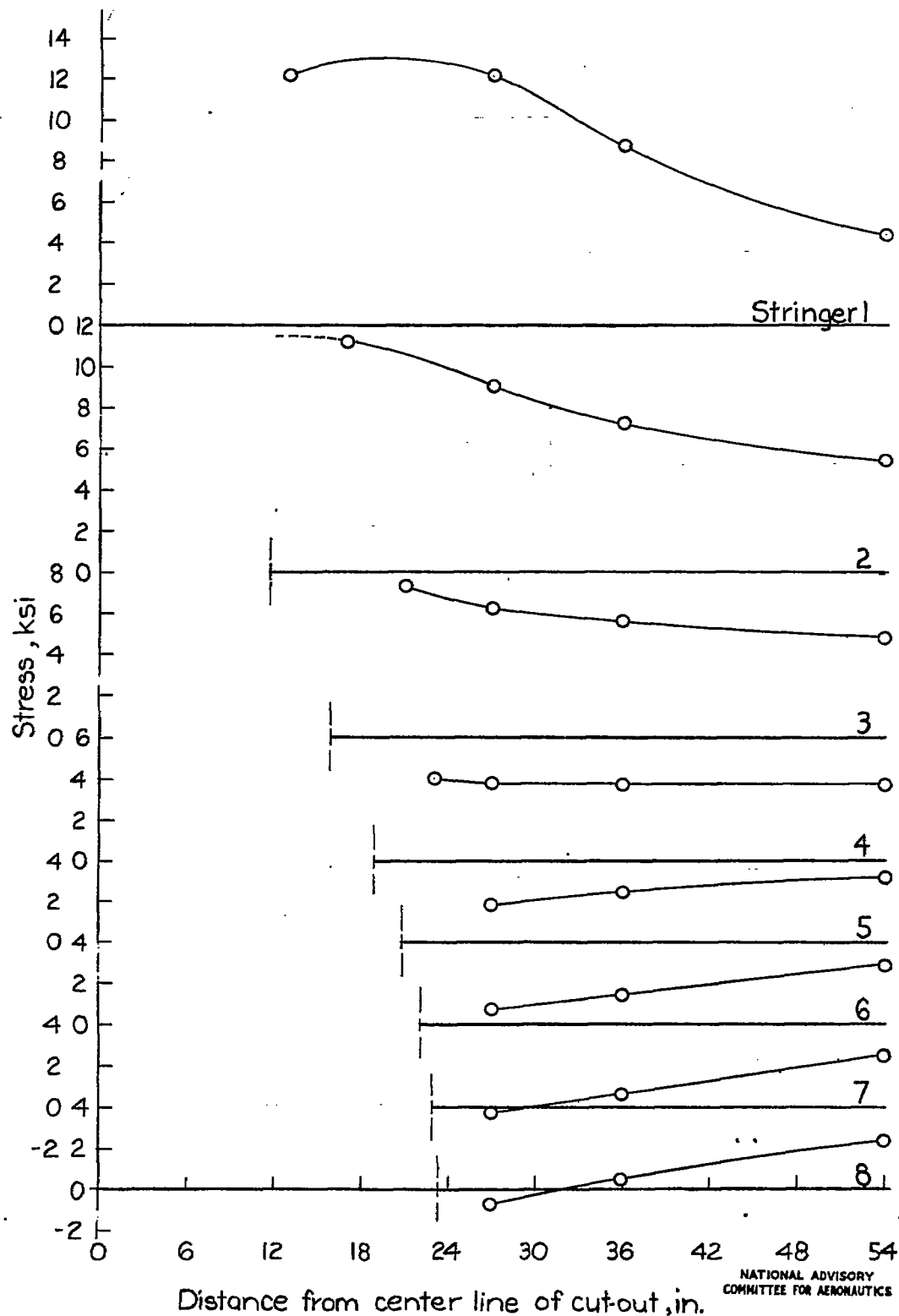Figure 4.-   Typical arrangement of wire strain gages on rings.

Figure 5.- Spanwise distribution of stringer stresses in panel with two
stringers cut. Dashed lines indicate extrapolation. Load = 15 kips.

Fig. 6   NACA TN No. 1241
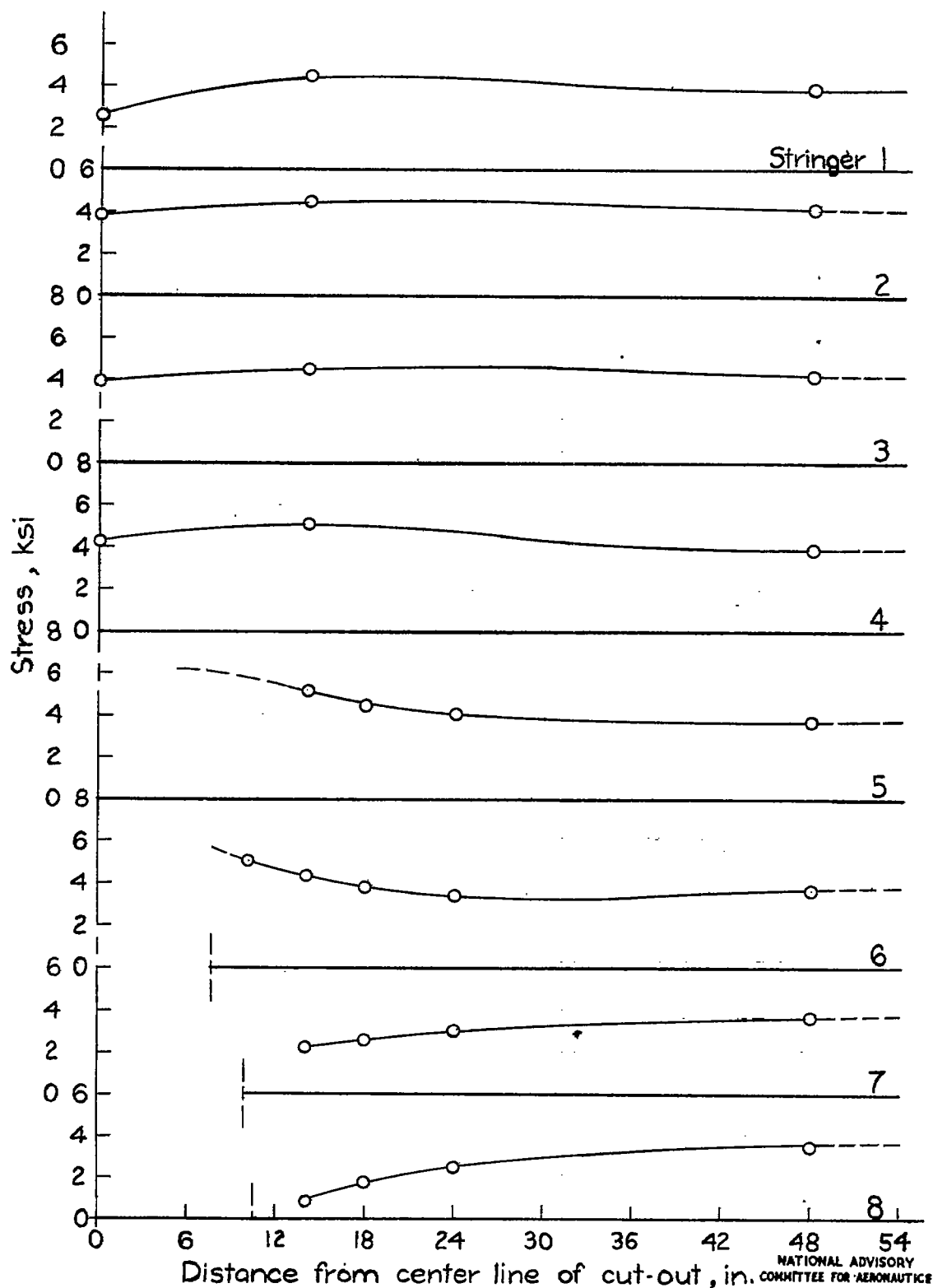
Figure 6.-   Spanwise distribution of stringer stresses in panel with six stringers cut.   Dashed lines indicate extrapolation.   Load = 15 kips.

Figure 7.- Spanwise distribution of stringer stresses in panel with ten stringers cut. Dashed lines indicate extrapolation. Load = 15 kips.
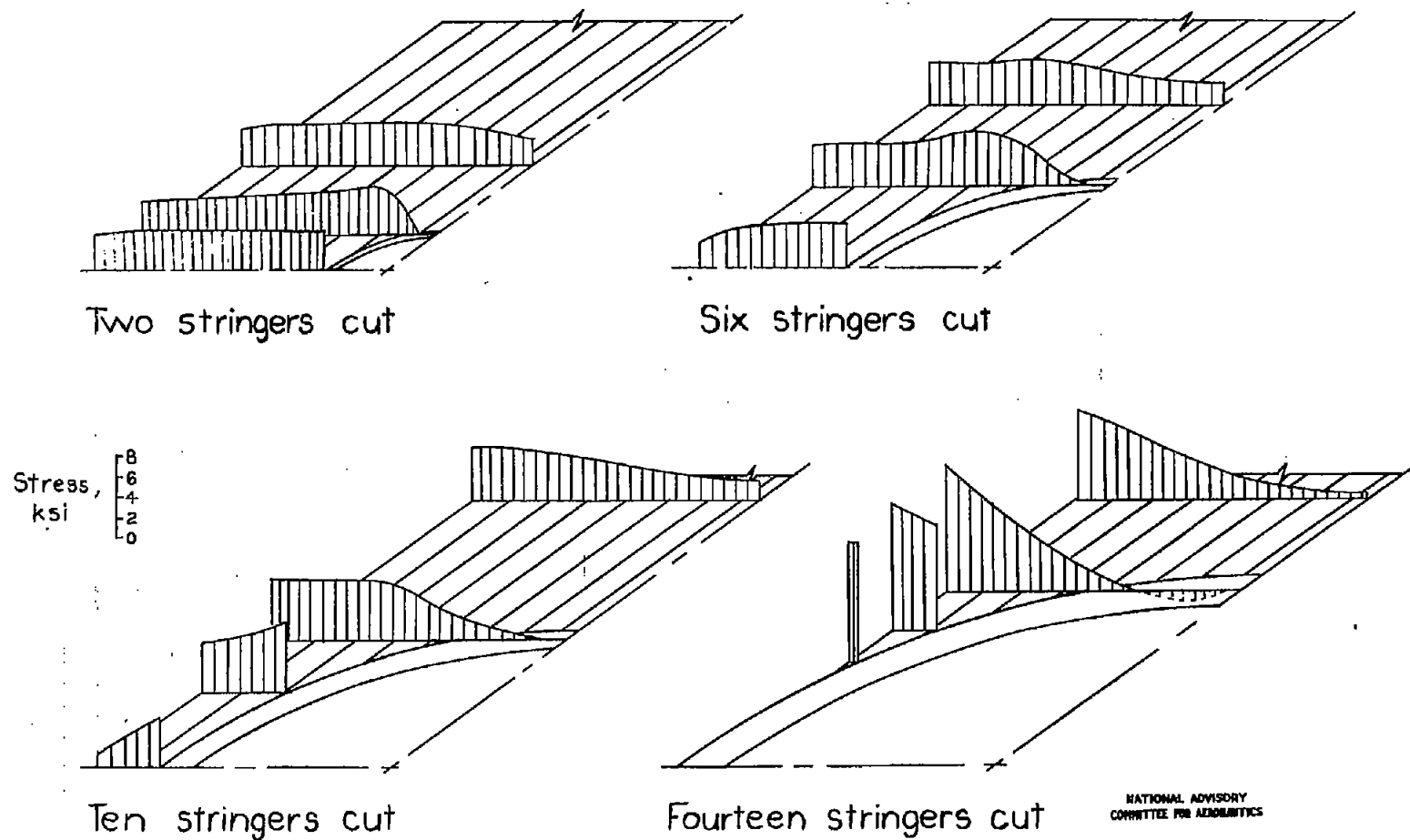
Fig. 8   NACA TN No. 1241

Figure 8.- Spanwise distribution of stringer stresses in panel with fourteen stringers cut. Dashed lines indicate extrapolation. Load = 15 kips.

Two stringers cut

Six stringers cut

Stress, ksi

$\begin{matrix}8\\6\\4\\2\\0\end{matrix}$

Ten stringers cut

Fourteen stringers cut

Figure 9.– Chordwise distribution of stringer stresses in one quadrant of panel. Load = 15 kips.

Fig. 9

Fig. 10   NACA TN No. 1241
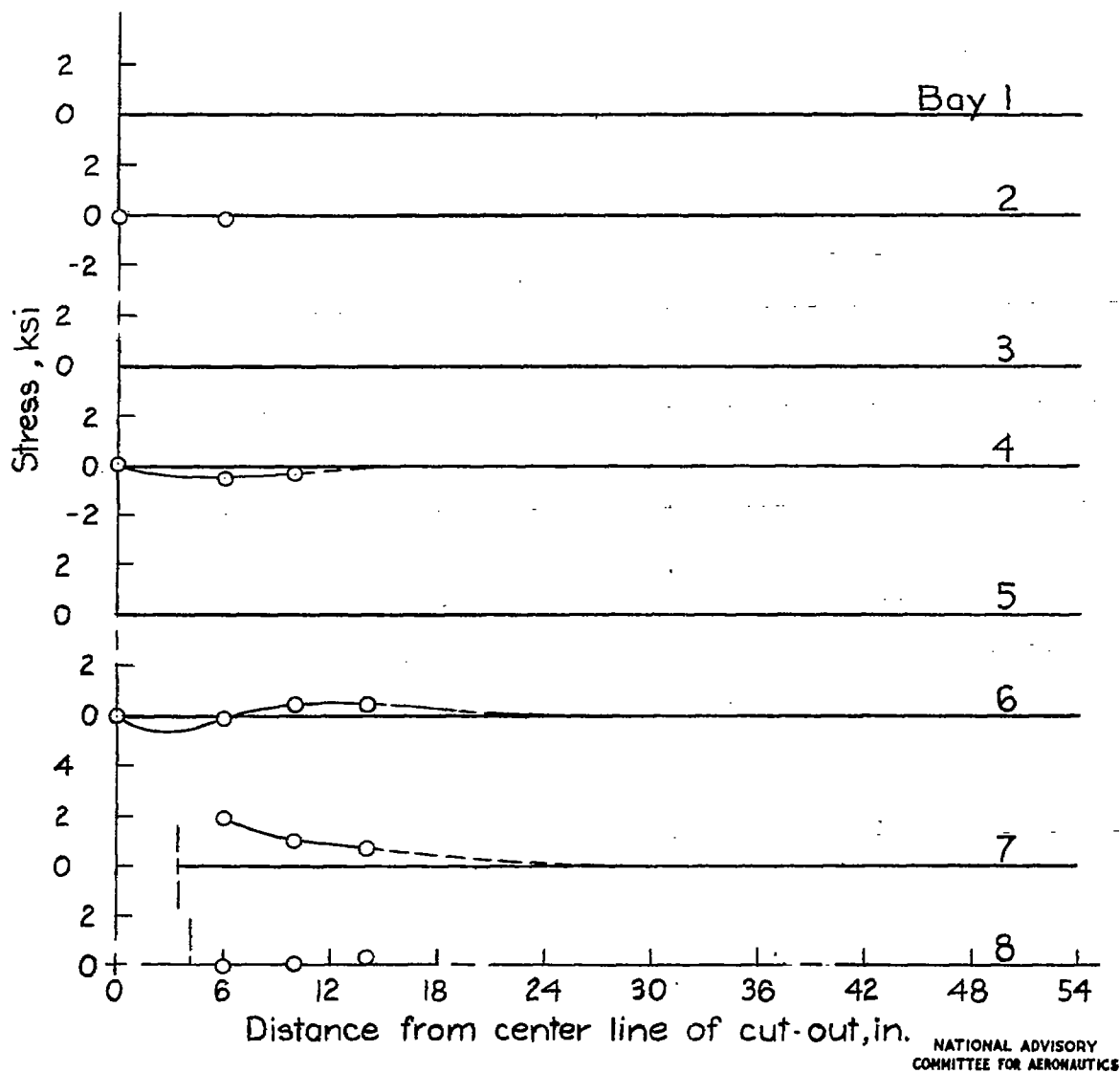
Figure 10.- Spanwise distribution of shear stresses in panel with two stringers cut. Load = 15 kips.

Figure 11.-   Spanwise distribution of shear stresses in panel with six
stringers cut.   Load = 15 kips.

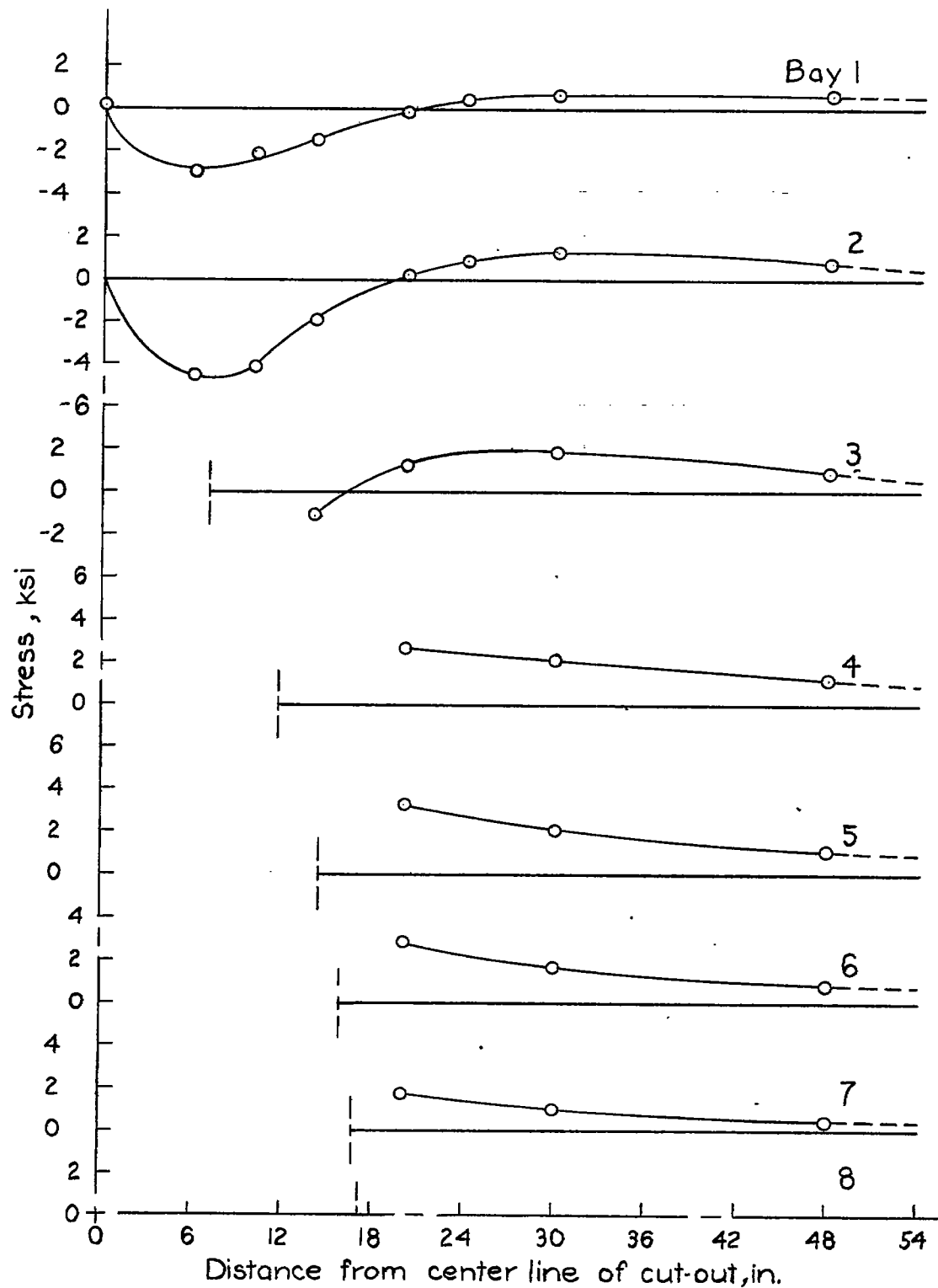Fig. 12   NACA TN No. 1241

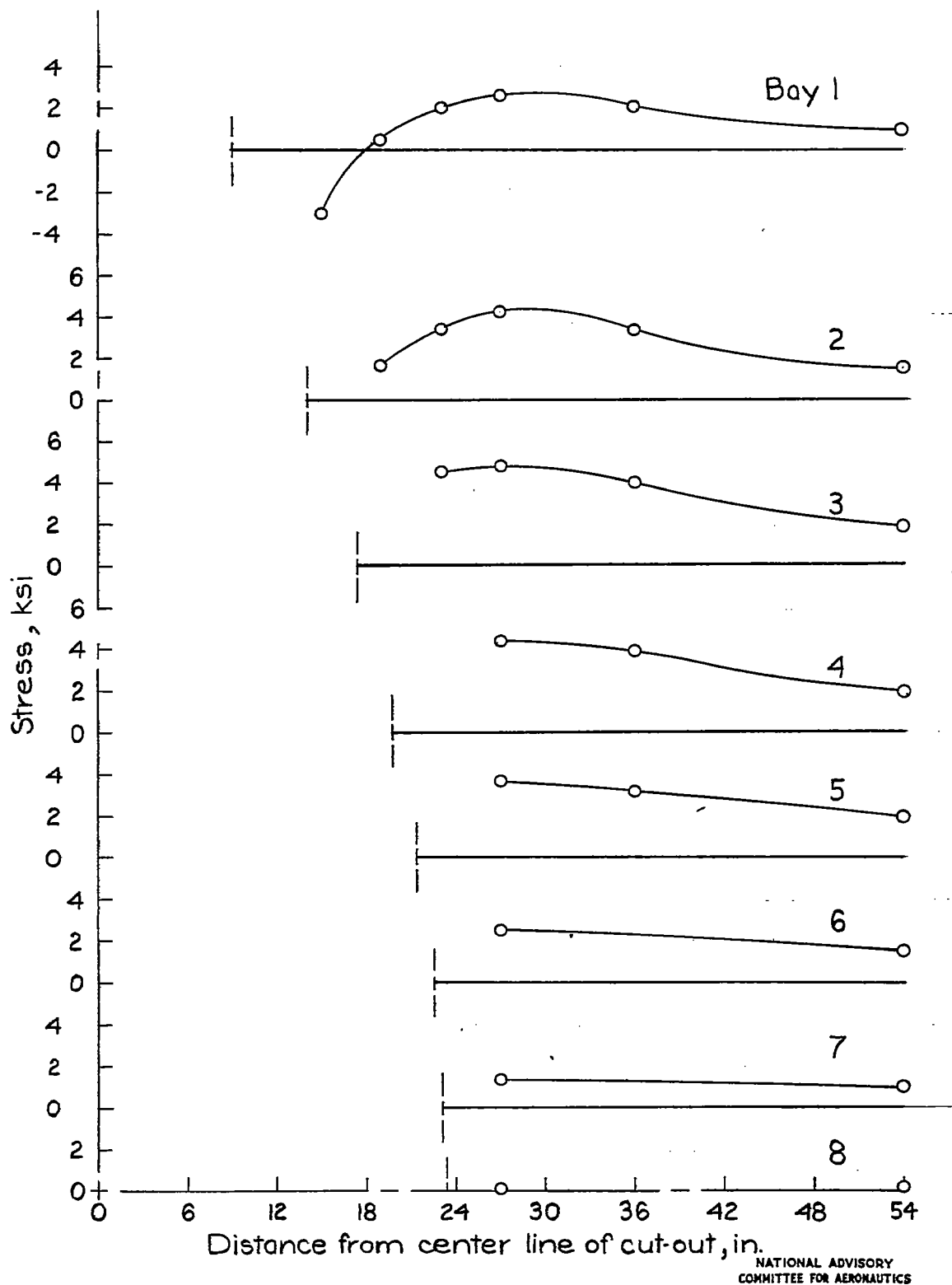Figure 12.- Spanwise distribution of shear stresses in panel with ten
stringers cut.   Load = 15 kips.

Fig. 13

Figure 13.- Spanwise distribution of shear stresses in panel with fourteen stringers cut. Load = 15 kips.

Fig. 14

Two stringers cut

Six stringers cut

Stress, ksi

Ten stringers cut

Fourteen stringers cut
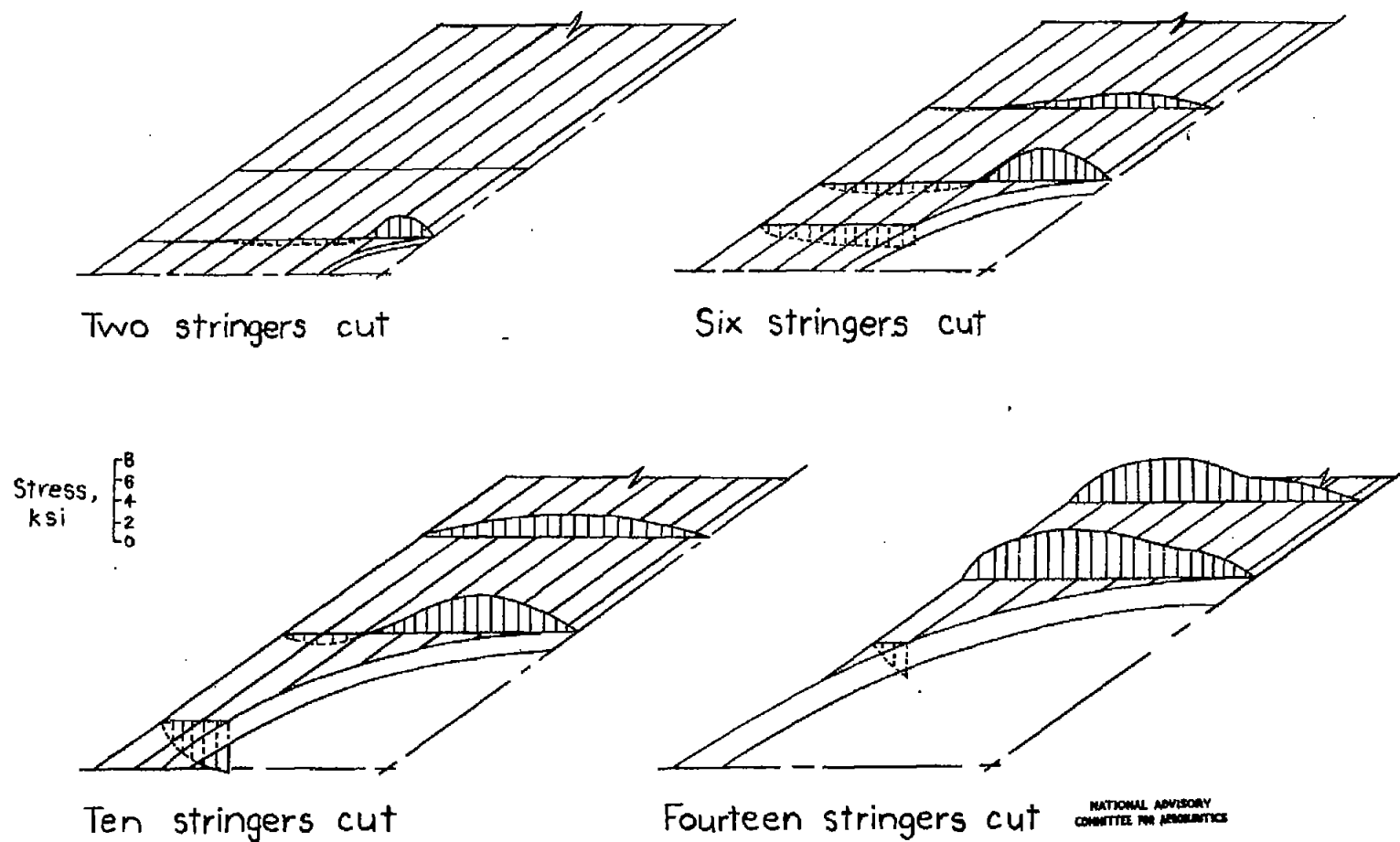
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Figure 14.- Chordwise distribution of shear stresses in one quadrant of panel. Load = 15 kips.
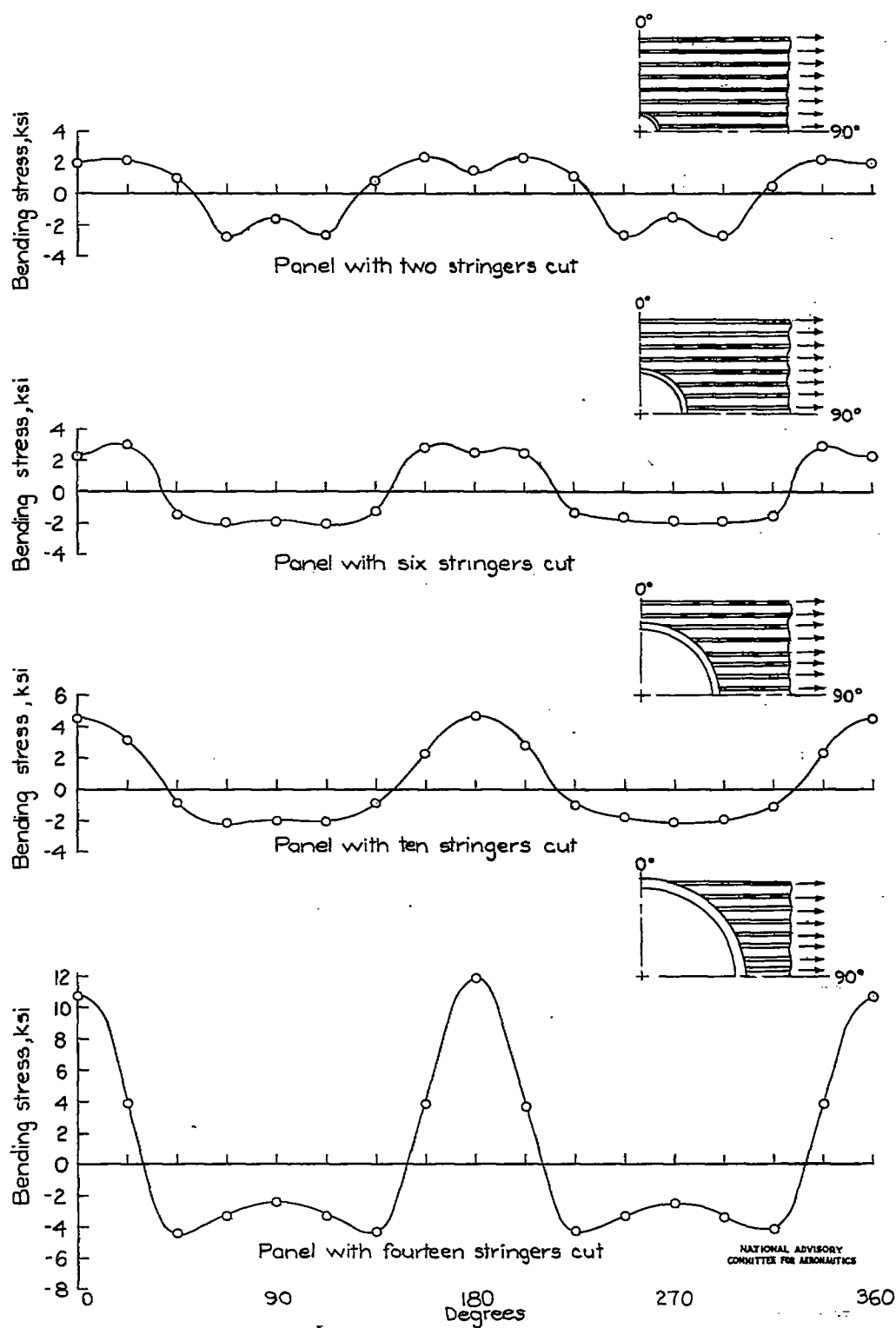
Figure 15.- Distribution of bending stresses in reinforcing rings.
   Load = 15 kips.